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# Model choice for phylogeographic inference using a large set of models

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## Abstract

Model-based analyses are common in phylogeographic inference because they parameterize processes such as population division, gene flow and expansion that are of interest to biologists. Approximate Bayesian computation is a model-based approach that can be customized to any empirical system and used to calculate the relative posterior probability of several models, provided that suitable models can be identified for comparison. The question of how to identify suitable models is explored using data from Plethodon idahoensis, a salamander that inhabits the North American inland northwest temperate rainforest. First, we conduct an ABC analysis using five models suggested by previous research, calculate the relative posterior probabilities and find that a simple model of population isolation has the best fit to the data (PP = 0.70). In contrast to this subjective choice of models to include in the analysis, we also specify models in a more objective manner by simulating prior distributions for 143 models that included panmixia, population isolation, change in effective population size, migration and range expansion. We then identify a smaller subset of models for comparison by generating an expectation of the highest posterior probability that a false model is likely to achieve due to chance and calculate the relative posterior probabilities of only those models that exceed this expected level. A model that parameterized divergence with population expansion and gene flow in one direction offered the best fit to the P. idahoensis data (in contrast to an isolation-only model from the first analysis). Our investigation demonstrates that the determination of which models to include in ABC model choice experiments is a vital component of model-based phylogeographic analysis.

*Keywords*: approximate Bayesian computation, demographic model selection, Pacific Northwest, *Plethodon idahoensis*, posterior predictive simulation

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# Introduction

Model-based analyses have become a common component of phylogeographic inference because they parameterize evolutionary processes that are of interest to biologists (Beaumont *et al.* 2010). To conduct a modelbased phylogeographic analysis, the choices available to researchers range from full likelihood implementations of predefined models to approximate methods that allow substantial customization of the model to the par-

Correspondence: Bryan C. Carstens, Fax: (614) 292-2030; E-mail: carstens.12@osu.edu ticulars of any empirical system. We prefer the latter option because it allows researchers to evaluate multiple demographic models relevant to their system, and to identify the model that offers the best fit to their data. In these cases, the process of model selection can guide phylogeographic inference by identifying the evolutionary processes (i.e. parameters) that have shaped the patterns of genetic variation (Carstens *et al.* 2013).

While model-based methods offer a number of benefits to phylogeographic investigations (Knowles 2009), the question of how researchers identify the models used to analyse their data is underexplored and can be a barrier to phylogeographic investigations. In model systems, results of prior research often guide the choice of analytical models (e.g. Smith *et al.* 2012). However, in nonmodel systems, there may be little beyond a basic understanding of life history to guide the choice of models to use in a phylogeographic analysis (e.g. Smith *et al.* 2011; Satler *et al.* 2013), and researchers are forced to rely on intuition to choose analytical methods. In these cases, the accuracy of inference is contingent on the fit of the assumed model to the empirical data. Because parameter estimates themselves are dependent to some degree on the model used (Koopman & Carstens 2010), it is likely that both parameter estimation and phylogeographic inference can be improved by incorporating phylogeographic model selection into the inference process.

In demographic model selection, phylogeographic inference is derived from a statistical comparison of multiple models given the data. For example, in a given system, there are clear implications to phylogeographic inference if an *n*-island model could be shown to be a much better fit to the data than a divergence with geneflow model. Phylogeographic model comparison can be conducted within some full likelihood programs, such as Migrate-n (Beerli & Palczewski 2010; Provan & Maggs 2012) or IMa2 (Hey & Nielsen 2007; Carstens et al. 2009), but these comparisons are limited to the set of models implemented within the respective programs. Model comparison is thus considerably more flexible when simulation-based approaches such as approximate Bayesian computation (ABC) are used. One of the earliest applications of ABC to the analysis of genetic data used this approach to demonstrate that a model of population growth was a better fit to human Y-chromosome data than a model without population growth (Pritchard et al. 1999). More recently, the approach has been used to compare competing models of human evolution (Fagundes et al. 2007; Laval et al. 2010) and to demonstrate that a model of a population bottleneck was a good fit to microsatellite data collected from chimpanzees (Peter et al. 2010).

Phylogeographic model selection using ABC is attractive for several reasons. First, while there are still technical challenges (e.g. computational intensity, choice of priors and summary statistics, and how to conduct the rejection step), phylogeographic model selection using ABC is conceptually simple (Beaumont 2010; Bertorelle *et al.* 2010; Csillery *et al.* 2011). It is conducted by generating a joint prior distribution from multiple models, forming a posterior distribution by selecting a small percentage of the simulated data that represents the closest match to the empirical data, then determining the relative contribution of each model to the posterior distribution. Second, ABC is flexible. The methods used to simulate the prior distribution are easily customized to nearly any empirical system and can be as complex or simple as desired; any model that can be simulated can be used in the analysis. While some authors have criticized phylogeographic model selection using ABC for ignoring differences in the complexity of models (i.e. the dimensionality as measured by number of parameters inherent to each model; Templeton 2010), the calculation of the marginal likelihood allows for differences in dimensionality across models (Beaumont et al. 2010), and thus, there is no need to correct for differences in the degree of parameterization. A more compelling criticism is related to the choice of the summary statistics used to summarize the simulated and empirical data. Robert et al. (2011) demonstrated that insufficient summary statistics could lead to a loss of information that can bias the calculation of the relative posterior probability, although they note that there are strategies for circumventing this difficulty (e.g. Ratmann et al. 2009: Sousa et al. 2009). Another criticism, and the factor that motivated this work, is related to the choice of the models to include in the analysis.

Phylogeographic model space is complex: there may be n subpopulations, the size of each could be described using an independent parameter  $\theta = 4N_e\mu$ , populations could be exchanging alleles at some rate Mij, each population could have diverged temporally from other populations at some time  $\tau$ , and each could be growing or expanding at some rate y. Our question is: 'How do researchers choose the models that they include in an ABC analysis?' Given the complexity of model space, it is impossible to generate the prior distribution to exhaustively cover hypothesis space represented by all possible models (Templeton 2009). On the surface, this is a general criticism to model-based methods, easily rebuked by alluding to the dictum of George Box: 'all models are wrong, but some are useful' (Box & Draper 1987). However, if model choice is used to guide phylogeographic inference (e.g. Fagundes et al. 2007) the pertinent question becomes, 'Are any of the models in our model comparison set useful?' Because the posterior probabilities are relative, the results could easily mislead researchers if the model set for comparison contains several wildly inappropriate models and one that is only a marginally better summary of the demographic history. As researchers who conduct investigations on nonmodel systems that typically lack prior information useful for model selection, this criticism is troubling. Furthermore, we suspect that this difficulty may be partially responsible for the reluctance of phylogeographers to broadly incorporate ABC into their investigations.

The goal of this study was to explore the fit of demographic models using ABC in *Plethodon idahoensis*, a terrestrial salamander from the Pacific Northwest (PNW) of North America. We take two approaches to identifying models to include in the analysis. First, we parameterize five phylogeographic models that have been used in previous work to see which is the best fit to our data. However, we have no a priori expectation that they represent models with a good fit to the empirical data so we also explore an objective approach to identifying demographic models. We consider 143 models that represent different combinations of the pertinent parameters (1 vs. 2 populations,  $\theta = 4Ne\mu$ , migration and population expansion) that could be used to describe demographic history in P. idahoensis. We rank each model according to the data using posterior predictive simulation (PPS) and develop a null expectation of the highest posterior probability that a false model can achieve by chance in the full 143 set of models. We then conduct a second model choice exercise using only those models that exceed this expectation. After exploratory analyses, we conclude that this objectively chosen model represents a better fit to the data collected from Plethodon idahoensis than does the best of the models used in previous studies.

#### Methods

#### Empirical data and study system

We use ABC analysis to explore the demographic history of *P. idahoensis*, the only *Plethodon* salamander located in the inland temperate rain forests of the northern Rocky Mountains of North America (Wilson & Larsen 1998). Previous work (Carstens *et al.* 2004) suggests that the dominant signal in genetic data is one of population expansion from southern refugia following glacial retreat at the end of the Pleistocene. However, the evidence for population structure within *P. idahoensis* is less clear. The fully terrestrial, lungless salamanders in



the genus Plethodon typically exhibit high site fidelity, small home range, defence of small territories and seldom disperse across habitats that expose them to dryness and heat (Smith & Green 2005). As a result, in a topographically diverse and geologically complex region like the PNW, terrestrial salamanders often reveal cryptic genetic diversity, even on geographic scales smaller than the widespread distribution of P. idahonesis (e.g. Mahoney 2004; Mead et al. 2005). In P. idahoensis, data from the mitochondrial genome indicate that there is population differentiation between the northern and southern river drainages. This structure is consistent with results from environmental niche modelling (Carstens & Richards 2007), and these findings prompted Carstens et al. (2009) to estimate demographic parameters using an isolation-with-migration model between these regions.

We gathered data from five genetic loci in 30 P. idahoensis individuals that were sampled throughout the range of the species in the northern and southern drainages (Fig. 1), thus expanding previous data sets. Samples from British Columbia are genetically identical to those from the northern portions of Idaho and Montana (Carstens et al. 2004) and not included here. Loci include the mitochondrial cytochrome b gene (Cyt b) and four autosomal loci: recombination activating gene 1 (RAG1), internal transcribed spacer ribosomal subunit 1 (ITS1), glyceraldehyde-3-phosphate dehydrogenase gene (GAPD) and an anonymous locus (Table 1). Loci exhibit no evidence of recombination using the fourgamete test or the SBP and GARD methods implemented in Hy-Phy (Pond & Frost 2005; Pond et al. 2006), and sequences generated for this study are deposited in GenBank under Accession nos JX978543-JX978577. Primer sequences and thermocycling conditions are available (Table S1, Supporting Information).

**Fig. 1** Sampling localities of *Plethodon idahoensis* in the Pacific Northwest, USA. Dark line is its distribution in Idaho, Montana and British Columbia. The dotted line within the distribution delineates the northern (population 2) and southern (population 1) river drainages. The large dotted line represents the extent of the ice sheets during the last glacial maximum.

Locus	Source	π	Segregating sites (SS)	Tajima's D (D)	π within pop1 (πW1)	π within pop2 (πW2)	π between pops (πB)
Cyt b	Carstens et al. 2004	0.0082	27	-0.9358	0.0047	0.0050	0.0113
RAG1	Weins et al. 2006	0.0024	13	-0.1791	0.0017	0.0020	0.0021
ITS1	Hillis and Dixon 1991	0.0019	4	-0.1810	0.0000	0.0023	0.0021
Gapd	Dolman and Phillips 2004	0.0032	8	-0.1153	0.0002	0.0036	0.0043
Anonymous	This study	0.0146	21	2.2552	0.0095	0.0160	0.0157

Table 1	Summary	statistics	for	5	loci
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Shown for each locus are the source of the primers, the length (bp) and six summary statistics: nucleotide diversity ( $\pi$ ), the number of segregating sites (SS), Tajima's *D* (*D*), nucleotide diversity within the northern ( $\pi$ W1) and southern ( $\pi$ W2) populations and nucleotide diversity between the northern and southern populations ( $\pi$ B).

Sanger sequencing was carried out with BigDye<sup>®</sup> TERMI-NATOR version 3.1 on an ABI 3130XL Genetic Analyzer (Applied Biosystems). Sequence editing and alignment were conducted using GENEIOUS version 5.4 (Drummond *et al.* 2011) and checked by eye. Sequence data were phased to alleles using PHASE (Stephens *et al.* 2001) with 95% confidence or were otherwise subcloned using the Qiagen PCR cloning kit. The GAPD locus included heterozygous indels so CHAMPURU version 1.0 (Flot 2007) was used to determine phase for some individuals. Six summary statistics ( $\pi$ , number of segregating sites, Tajima's *D*,  $\pi$  within each of the northern and southern populations and  $\pi$  between populations) were calculated for each locus using DnaSP (Rozas *et al.* 2003).

#### Phylogeographic models and summary statistic testing

ABC was utilized for several analyses (below). Prior distributions for 143 demographic models were simulated using the program ms (Hudson 2002), with data simulated to match the number of chromosomes sampled under each locus, and simulations were scaled to correspond to the mitochondrial locus. Prior distributions consisted of 100 000 simulated data sets for each of the 143 demographic models. Demographic models were defined on the basis of four categories of parameters: (i) models were defined as either n-island, divergence from a common ancestor, or panmixia; (ii)  $\theta = 4N_e\mu$  (N<sub>e</sub> is the effective population size and  $\mu$  is the per-locus mutation rate) was either the same in all populations at all time periods, unique in all populations at all time periods or the same in some combination of populations at some time periods; (iii) migration was either not included, present in both directions between populations 1 and 2, or in one direction only; (iv) population expansion was either not included, included in one population or included in both populations (Fig. 2). A PERL script (available at doi: 10.5061/ dryad.8kq65) was used to draw values from uniform prior distributions for the parameters ( $\tau$ ,  $\theta$ ,  $\theta_1$ ,  $\theta_2$ ,  $m_{12}$ ,  $m_{21}$ ,  $\gamma_1$  and/or  $\gamma_2$ ) present in each model and used to simulate the genealogies. The upper and lower bounds

Divergence time $(\tau)$	Theta (0)	Migration (m)	Population expansion (y)
0: island model 1: divergence at time (τ) X: pamixia	$0: \theta_{A} = \theta_{1} = \theta_{2}$ $1: \theta_{A} = \theta_{1}, \theta_{2}$ $2: \theta_{A} = \theta_{2}, \theta_{1}$ $3: \theta_{A}, \theta_{1} = \theta_{2}$ $4: \theta_{A}, \theta_{1}, \theta_{2}$	0: no migration 1: m <sub>12</sub> 2: m <sub>21</sub> 3: m <sub>12</sub> , m <sub>21</sub> X: na/pamixia	0: no expansion 1: $\gamma_1$ 2: $\gamma_2$ 3: $\gamma_1$ , $\gamma_2$
Prior: 0.001–5 (4N generations)	Prior: 0.01–10 per locus	Prior: 0–5 migrants per generation	Prior: 0.1–9 (exponential)

#### For each model: τθmy

Fig. 2 Numeric coding for demographic models being tested. All models are identified with a 4-digit number that describes the parameter combination associated with a particular model corresponding to population divergence,  $\theta = 4N_{em}$ , migration rates and the extrinsic rate of population expansion. Each parameter can be assigned to the northern (subscript 2) or southern (subscript 1) populations in one of several combinations. Uniform priors are based on previous *Plethodon* work and occupy the full range of biologically reasonable values and are according to ms (Hudson 2002) documentation.

of parameters included in a given model were derived from previous analyses in these salamanders ( $\theta_{\text{locus}} = 0.01-10.0$ ;  $\tau = 0.001-5.0$ ; m = 0-5.0;  $\gamma = 0.01-9.0$ ) to cover the range of biologically plausible values for each parameter given our system. Summary statistics from simulated data ( $\pi$ , number of segregating sites, Tajima's D,  $\pi$  within each of the northern and southern populations and  $\pi$  between populations) were calculated using a custom PERL script written by (N. Takebayashi, personal communication).

The six summary statistics collected from the data were calculated for all simulations, and 24 combinations of these summary statistics were evaluated to determine which vector of summary statistics maximized the probability of choosing the true model. For each of the models, 10 data sets were selected at random from the prior distribution as pseudo-empirical data sets (a total of 1430 tests) for the ABC rejection step in msBayes (Hickerson et al. 2007). In order to choose the most appropriate vector of summary statistics for the identification of demographic scenarios (Marin et al. 2011; Robert et al. 2011), vectors were ranked according to their ability to maximize the probability of choosing the true model over the average probability of choosing an incorrect model (Pr<sub>(true model)</sub>/mean Pr<sub>(false models</sub>); Tsai & Carstens 2013). After simulation testing, one vector of summary statistics was chosen for use in all subsequent ABC analyses (see Table S2, Supporting Information).

#### Approximate Bayesian computation

After a series of trials exploring threshold size and the utility of regression-based corrections (Table S3, Supporting Information), a simple rejection step was conducted using msBayes (Hickerson *et al.* 2007) and a threshold size of 0.0002–0.00005 was chosen to retain 100–715 models in the posterior for model prior sets containing between 5 and 143 models. We conducted several ABC analyses:

1 We compared five models that were either inferred or assumed in previous investigations (Fig. 3). These models include that of a single panmictic population, an expansion from a single refuge (Carstens *et al.*  2004), an isolation model with no size change (Carstens & Richards 2007), an isolation model with size change between the ancestral and descendant populations (Carstens *et al.* 2009) and a full isolation-with-migration model and size change between the ancestral and descendant populations (Carstens *et al.* 2009). We also calculated Bayes factors (BF) to evaluate the strength of evidence (Kass & Raftery 1995) in favour of the model with the highest posterior probability.

- 2 We randomly selected four models from the full set of models and included the best model from the empirical comparison above. In this way, we generated prior distributions from 100 replicated model sets (each containing five demographic models) intended to allow us to visualize the influence of model set composition on the relative posterior probability (PP) of the best (as chosen above) model. The rejection step outlined above was used to generate a relative PP of the chosen model for each replicate. While this approach does not truly replicate the analysis (because the composition of the prior distribution differs in each replicate), it illustrates the influence of the set of models in the prior distribution on the PP of the model identified as optimal in the initial analysis.
- 3 We conducted simulation testing to evaluate the ability of ABC to identify the model used to generate the data relative to the number of models included in the prior distribution. We anticipate that the accuracy of ABC in regard to identifying the true model will decrease as a function of the number of models included in the comparison because the prior probability of each model is a function of the total number of models in the comparison (from 0.5 in a comparison of two models to approximately 0.007 in our comparison of all 143 models) and there are more ways to be incorrect as the number of models increases. The simulation study will test this expectation, but can also be used to generate an expectation for the highest PP that a false model could have by chance. To do this, we randomly selected a set of models equivalent in size to the posterior distribution



**Fig. 3** Diagram of the five demographic models (Carstens *et al.* 2004, 2009; Carstens & Richards 2007) used for the first round 5-model ABC analysis. Each model includes one or more parameters, is labelled by a numeric code and includes its posterior probability in the initial ABC model choice analysis.

of a given trial and calculated over 100 replicates the number of times that the most-represented model occurred.

- **4** We conducted a single ABC analysis using all of the models in the comparison set, for a total of 143 demographic scenarios. While we are not exploring every conceivable model in this approach, we do include relevant parameters based on prior knowledge of the *P. idahoensis* populations and thus these models serve as a representative sample of possible models that either treat the northern and southern drainages as the same or as distinct populations.
- **5** We explored the influence of different classes of parameters by grouping models from the posteriors and comparing them as follows: (i) island vs. panmictic vs. isolation; (ii) no change in  $\theta$  vs. change in  $\theta$  in population 1 vs. change in  $\theta$  in population 2 vs. change in  $\theta$  in both populations; (iii) migration from population 2 to 1 vs. migration from population 1 to 2 vs. migration in both directions vs. no migration; and (iv) expansion in population 1 vs. expansion in population 2 vs. no population expansion. The total PP for each group was determined from the 143-model ABC test for each group.
- **6** Finally, the mean and 95% confidence intervals of all parameters were estimated using R 2.15.1 (R Core Team 2012) using a select pair of models (below).

#### Posterior predictive simulation

In addition to the ABC analyses, we calculated the fit of models in a nonrelative way to the empirical data using posterior predictive simulation (PPS; Gelfand & Ghosh 1998; Cornuet *et al.* 2010; François & Laval 2011). This was done by calculating the mean Euclidean distance (MED) of the vector of all estimated summary statistics from simulated data under each of the 143 models to the empirical data. We also plotted the PPS distributions of individual summary statistics used in the ABC analyses for three of the models to explore model adequacy and identify any bias in summary statistics.

To conduct the PPS, the rejection step in msBayes (Hickerson *et al.* 2007) was incorporated into a pipeline and used to generate a posterior distribution for each model (threshold = 0.0005 to retain 50 models in the posterior). These data points represent the simulated data sets closest to the empirical data for each model based on our chosen vector of summary statistics. The PPS used each point in the posterior distribution to simulate 100 new genealogies and associated summary statistics, so the distribution from the PPS contained a

total of 5000 points. Thus, the variation in the genealogies (and associated summary statistics) is assessed based on specific demographic parameter values for any given model that represent those closest to the empirical data. As Euclidean distance is used for the ABC rejection step, we chose this measure to rank the distance of the models to the *P. idahoensis* data rather than plotting each summary statistic, although this was done for 15 summary statistics for three models (see above). From the PPS distribution, we calculated the MED from the empirical data to each point in the simulated data, and this distance was used to measure the nonrelative fit of the model to the *P. idahoensis* data.

# **Results & discussion**

#### Empirical data and summary statistics

Sequence data were gathered for five loci in 30 samples. Summary statistics ( $\pi$ , number of segregating sites, Tajima's D,  $\pi$  within each of two populations and  $\pi$ between populations) for each locus are shown in Table 1. Prior distributions for each model were drawn from the prior range of associated parameter values ( $\tau$ ,  $\theta$ ,  $\theta_1$ ,  $\theta_2$ ,  $m_{12}$ ,  $m_{21}$ ,  $\gamma_1$ ,  $\gamma_2$ ; see Fig. 2), and summary statistics were generated from each of 100 000 draws. After simulation testing, the summary statistic vector { $\pi$ within population 1,  $\pi$ within population 2,  $\pi$ between populations} maximized Pr(true model)/mean Pr(false models) (Table S2, Supporting Information) and was used for all ABC analyses because it chose the correct model with greater accuracy than the other vectors.

### Approximate Bayesian computation

We first conducted an ABC analysis using five models suggested by previous research (Fig. 3) and determined that the isolation model without gene flow or change in  $\theta$  (model designated 1000 in our numeric labelling scheme; Fig. 2) had the highest PP (0.70). While two other isolation models had some posterior support, the models without subdivided populations were not represented in the posterior. The posterior support in favour of model 1000 is modest (BF =  $\sim$ 4.7) under the Kass & Raftery (1995) scale, but the parameter estimates from this model are reasonable (Table 2). Using the nDNA and assuming a neutral mutation rate of  $1.0 \times 10^{-9}$ substitutions/site/generation and our average sequence length of 561, the effective population size of P. idahoensis would be approximately 34 700 individuals and the temporal divergence between the populations dates to the mid-Pleistocene (approximately 218 000 generations). On the surface, the choice of this model appears biologically plausible because these parameter estimates

Model	θ	$\theta_1$	$\theta_2$	τ	$m_1$	m <sub>2</sub>	$\gamma_1$	γ <sub>2</sub>
1000	0.079 (0.059–0.097)	na	na	1.569 (1.176–1.961)	na	na	na	na
1023	0.089 (0.061 to 0.117)	na	na	1.933 (1.507 to 2.359)	na	2.858 (2.349 to 3.368)	3.776 (-0.668 to 8.220)	1.678 (0.720 to 2.635)

Table 2 Population parameter estimates

Estimates are from mean values in posterior distributions. 95% CI are shown in parentheses below the estimates. Models that lack a given parameter are marked with an 'na'.

confirm previous expectations. However, because we were curious about how the relative PP of this model is influenced by the choice of models in the comparison set, we conducted an ABC analysis with models selected at random to compare with this best model (1000).

We randomly selected four models from the set of 142 possible models (i.e. all but model 1000) and repeated this five-model analysis 100 times. Results demonstrate that model set composition is an important consideration in ABC model choice exercises (Appendix S1 Supporting Information); model 1000 had a mean PP across replicates of  $PP_{1000} = 0.44$  with a wide range (0.21–0.77). This illustrates the inherent challenge to ABC model choice; depending on the models chosen in the comparison set, relative posterior support could favour or oppose a given model to a degree that would appear meaningful based on traditional interpretations of the PP. This result also raises questions that are either specific to our data (i.e. How much information are contained in our data? Are the collected data adequate to identify the true model in a comparison of five models?) or general to ABC (i.e. Are we including models that accurately represent our data? Does the probability of selecting the true model change as a function of the number of models included in the analysis?). We addressed the question about the information contained in our data by simulating additional loci based on the averaged characteristics of the empirically sampled data, and found that increasing the amount of data collected from 5 to 20 loci did not substantially improve our ability to differentiate models (Appendix S2, Supporting Information). While this is not an exhaustive analysis, it does indicate that a 4-fold increase in the amount of data has a negligible effect on the power of the analysis to differentiate models. Therefore, we expanded the simulation study to address the more general questions.

A power analysis was conducted to explore the relationship between the accuracy in identifying the true model and the number of models that contribute to the prior distribution (Fig. 4). The number of models (*n*) varied from 2, 3, 4–20 (increments of 2), 30–130 (increments of 20) and 143. We analysed 100 replicated data sets, with the true model and n-1 additional models chosen at random for each replicate. Results indicate that ABC performs well (measured by average PP of the true model) when a small number of models contribute to the prior distribution, but that accuracy quickly decreases to just above the prior probability above n = 4. When we generated an expectation of the average probability of the most-represented model found in a random sample of models of the same size



# Simulation testing results

Fig. 4 Simulation testing to investigate the performance of ABC model choice as a function of the number of models. The prior probability (blue line), averaged probability of the most-represented model in a selection equal to the size of the posterior (red line), posterior probability of the true model (green line) and the median Bayes factor (dotted line) are shown. of the posterior at a given increment of the power analysis, we found that this number exceeded the PP of the true model above n = 3. Therefore, the expected PP of the true model decreases as a function of how many models are included in the model choice experiment. However, the generation of this random expectation allows us to identify a smaller set of models that can be identified because they exceed the random expectation. In many cases, this smaller set includes the true model as well as similar models, because as the number of models in the prior distribution increases, the difference among these models decreases, resulting in a posterior distribution that contains both the true model and a set of models that are similar to it.

To explore this suggestion, we conducted a large analysis using the empirical data and prior distributions from all (i.e. 143) of the models used above in the simulation testing. After the rejection step, 120 models were represented in the posterior distribution (Fig. 5; Table 3). As anticipated, the posterior probabilities for all models were low, although many were greater than the prior expectation of approximately 0.007. The models with the highest PP in this analysis included a mix of isolation and island models, and within these categories, the parameterization was similar. None of the models represented in the posterior above the random threshold parameterize a change in  $\theta$ , while most included some sort of gene flow and expansion in at least one of the populations. The similarity of these models explains why model choice experiments with ABC decrease in accuracy as the number of models increases; as the parameterization of models become more similar, the posteriors are populated by models that are similar to the true model. Notably, the model

chosen as best in the initial 5-model test had a PP (0.015) lower than the random expectation (0.016) in the 143 model analysis. This result is robust to change in the size of the threshold used in the rejection step, and nearly the same when regression (Beaumont 2010) is used in model selection (see Table S3 & Appendix S1, Supporting Information), further suggesting that model 1000 is not representative of the demographic history of *P. idahoensis*. Consequently, we focus on the models that occur in the posterior in proportions that are greater than expected at random (Table 4).

In P. idahoensis, approximately 1/6 of the models (22 of 143) were represented in the posterior distribution of the full analyses at greater than random (>0.016) levels. As most of the models had some type of gene flow and expansion, but were either isolation or island models, we divided (following Fagundes et al. 2007) this set into two groups (island and isolation), conducted another rejection step in each and compared the best models. After model comparison within each category, models 0033 and 1023 were retained with the highest PP among the island and isolation models, respectively (Table 4). When these two models were compared directly, the isolation model (1023) was substantially better  $(PP_{1023} > 0.9, depending on the threshold size), clearly$ indicating that temporal divergence between the northern and southern populations should be modeled. However, it is also clear that within each set (i.e. the island and isolation models), a number of models are very similar in their support. While this does not influence the results of the one-to-one comparison of isolation models to island models (BF > 9 for comparison of either 1021, 1033 or 1032 to 0033), it does suggest that we are limited in our ability to differentiate among the



**Fig. 5** Results from 143-model ABC analysis. Black bars are relative posterior probabilities. Dotted line is the prior probability. Solid line is the average highest PP observed by random chance. Only the 5-model test models and models with the highest and lowest PP are labelled for clarity. All model PP and MED values are in Table 3.

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Model	Parameters	Mean	SD	Median	Posterior probability
1030	$\tau, \ \theta_A = \theta_1 = \theta_{2,} \ m_{12,} \ m_{21}$	0.792	1.124	0.000	0.024
1232	$\tau, \ \theta_A = \theta_{2,} \ \theta_{1,} \ m_{12,} \ m_{21,} \ \gamma_2$	0.822	0.856	0.772	0.007
1200	$\tau$ , $\theta_A = \theta_{2}$ , $\theta_1$	0.836	0.985	0.499	0.004
1222	$\tau, \ \theta_A = \theta_{2,} \ \theta_{1,} \ m_{21,} \ \gamma_2$	0.846	0.982	0.542	0.006
1220	$\tau, \ \theta_A = \theta_{2,} \ \theta_{1,} \ m_{21}$	0.849	0.957	0.647	0.006
1231	$\tau, \ \theta_{A} = \theta_{2,} \ \theta_{1,} \ m_{12,} \ m_{21,} \ \gamma_{1}$	0.863	0.877	0.859	0.006
1221	$\tau, \ \theta_A = \theta_2, \ \theta_{1,} \ m_{21,} \ \gamma_1$	0.870	0.878	0.862	0.011
1031	$\tau, \ \theta_{\mathrm{A}} = \theta_1 = \theta_{2}, \ m_{12}, \ m_{21}, \ \gamma_1$	0.886	1.133	0.000	0.020
1230	$\tau, \ \theta_A = \theta_{2,} \ \theta_{1,} \ m_{12,} \ m_{21}$	0.917	0.937	0.880	0.006
1033	$\tau, \ \theta_{A} = \theta_{1} = \theta_{2}, \ m_{12}, \ m_{21}, \ \gamma_{1}, \ \gamma_{2}$	0.923	1.170	0.000	0.018
0131	$\theta_A=\theta_{1,}\;\theta_{2,}\;m_{12,}\;m_{21,}\;\gamma_1$	0.930	1.024	0.779	0.007
0130	$\theta_A = \theta_{1,} \ \theta_{2,} \ m_{12,} \ m_{21}$	0.949	0.881	1.055	0.010
1023	$\tau, \ \theta_A = \theta_1 = \theta_{2,} \ m_{21,} \ \gamma_{1,} \ \gamma_2$	0.956	1.154	0.000	0.024
1201	$\tau,  \theta_A = \theta_2,  \theta_1,  \gamma_1$	0.975	1.026	0.866	0.006
0030	$\theta_{A} = \theta_{1} = \theta_{2,} \ m_{12,} \ m_{21}$	0.977	1.210	0.000	0.024
1211	$\tau, \ \theta_A = \theta_2, \ \theta_1, \ m_{12}, \ \gamma_1$	0.990	1.042	0.927	0.007
0020	$\theta_{\mathrm{A}} = \theta_1 = \theta_{2,} \ \mathrm{m}_{12,} \ \mathrm{m}_{21}$	0.991	1.264	0.000	0.017
1132	$\tau, \ \theta_A = \theta_1, \ \theta_2, \ m_{12}, \ m_{21}, \ \gamma_2$	0.995	0.981	0.986	0.007
0031	$\theta_{\mathrm{A}} = \theta_1 = \theta_{2,} \ \mathbf{m}_{12,} \ \mathbf{m}_{21,} \ \gamma_1$	0.996	1.303	0.000	0.020
0022	$\theta_{\rm A} = \theta_1 = \theta_2, \ m_{21}, \ \gamma_2$	1.003	1.241	0.000	0.025
1131	$\tau, \ \theta_A = \theta_{1,} \ \theta_{2,} \ m_{12,} \ m_{21,} \ \gamma_1$	1.011	0.967	1.013	0.004
1032	$\tau, \theta_{\rm A} = \theta_1 = \theta_{2,} m_{12,} m_{21,} \gamma_2$	1.013	1.212	0.000	0.031
1212	$\tau, \ \theta_A = \theta_2, \ \theta_{1,} \ m_{12}, \ \gamma_2$	1.015	0.986	1.083	0.003
1233	$\tau, \ \theta_A = \theta_2, \ \theta_1, \ m_{12}, \ m_{21}, \ \gamma_1, \ \gamma_2$	1.021	0.946	1.121	0.010
1203	$\tau$ , $\theta_{\rm A} = \theta_{2}$ , $\theta_{1}$ , $\gamma_{1}$ , $\gamma_{2}$	1.024	1.058	1.002	0.010
0233	$\theta_{A} = \theta_{2},  \theta_{1},  m_{12},  m_{21},  \gamma_{1},  \gamma_{2}$	1.026	0.985	1.118	0.004
1110	$\tau, \ \theta_A = \theta_{1,} \ \theta_{2,} \ m_{12,} \ \gamma_1$	1.030	1.003	1.118	0.007
0222	$\theta_{A} = \theta_{2,} \ \theta_{1,} \ m_{21,} \ \gamma_{2}$	1.031	1.112	0.921	0.008
1130	$\tau, \ \theta_A = \theta_{1,} \ \theta_{2,} \ m_{12,} \ m_{21}$	1.031	0.976	1.084	0.006
0112	$\theta_A = \theta_{1_{\prime}} \; \theta_{2_{\prime}} \; m_{12_{\prime}} \; \gamma_2$	1.032	0.991	1.121	0.007
0032	$\theta_{\mathrm{A}} = \theta_1 = \theta_{2}, \ \mathbf{m}_{12}, \ \mathbf{m}_{21}, \ \gamma_2$	1.033	1.212	0.000	0.020
0110	$\theta_A = \theta_{1_{\prime}} \; \theta_{2_{\prime}} \; m_{12_{\prime}} \; \gamma_1$	1.034	1.031	1.070	0.004
1020	$\tau, \; \theta_{A} = \theta_{1} = \theta_{2} \; m_{12} \; m_{21} \; \gamma_{1} \; \gamma_{2}$	1.035	1.196	0.000	0.015
0012	$\theta_{\rm A} = \theta_1 = \theta_{2\prime} \ m_{12\prime} \ \gamma_2$	1.038	1.272	0.000	0.018
1213	$\tau, \ \theta_A = \theta_2 = \theta_1, \ m_{12}, \ \gamma_1, \ \gamma_2$	1.041	1.053	1.121	0.003
0220	$\theta_{\rm A} = \theta_2, \ \theta_1, \ m_{21}$	1.041	0.965	1.121	0.010
1013	$\tau$ , $\theta_A = \theta_1 = \theta_2$ , $m_{12}$ , $\gamma_1$ , $\gamma_2$	1.042	1.227	0.543	0.024
0231	$\theta_A = \theta_2, \ \theta_1, \ m_{12}, \ m_{21}, \ \gamma_1$	1.048	1.104	0.997	0.007
1111	$\tau$ , $\theta_A = \theta_1$ , $\theta_2$ , $m_{12}$ , $\gamma_1$	1.050	1.027	1.098	0.013
0013	$\theta_{A} = \theta_{1} = \theta_{2}, \ m_{12}, \ \gamma_{1}, \ \gamma_{2}$	1.056	1.254	0.000	0.021
0133	$\theta_{\rm A} = \theta_1,  \theta_2,  m_{12},  m_{21},  \gamma_1,  \gamma_2$	1.057	1.107	1.028	0.001
0033	$\theta_{A} = \theta_{1} = \theta_{2}, \ m_{12}, \ m_{21}, \ \gamma_{1}, \ \gamma_{2}$	1.059	1.289	0.000	0.031
1002	$\tau$ , $\theta_A = \theta_1 = \theta_2$ , $\gamma_2$	1.084	1.261	0.000	0.008
1331	τ, $θ_A$ , $θ_1 = θ_2$ , $m_{12}$ , $m_{21}$ , $γ_1$	1.098	1.093	1.081	0.000
0132	$\theta_A = \theta_{1_{\prime}} \theta_{2_{\prime}} m_{12_{\prime}} m_{21_{\prime}} \gamma_2$	1.101	0.991	1.129	0.007
0210	$\theta_A = \theta_{2} \theta_{1} m_{12}$	1.102	1.111	1.040	0.001
1321	$\tau, \ \theta_{A_{\prime}} \ \theta_{1} = \theta_{2_{\prime}} \ m_{21_{\prime}} \ \gamma_{1}$	1.108	1.012	1.124	0.000
1123	$\tau, \ \theta_A = \theta_1, \ \theta_2, \ m_{21}, \ \gamma_1, \ \gamma_2$	1.118	1.094	1.121	0.003
1021	$\tau$ , $\theta_A = \theta_1 = \theta_{2}$ , $m_{21}$ , $\gamma_1$	1.119	1.323	0.000	0.036
1113	$\tau, \theta_{\rm A} = \theta_1, \theta_2, m_{12}, \gamma_1, \gamma_2$	1.132	1.042	1.129	0.003
1010	$\tau$ , $\theta_A = \theta_1 = \theta_2$ , $m_{12}$	1.135	1.284	0.558	0.013
1112	$\tau$ , $\theta_A = \theta_1$ , $\theta_2$ , $m_{12}$ , $\gamma_1$	1.135	0.943	1.137	0.006
1101	$\tau, \theta_{A} = \theta_{1}, \theta_{2}, \gamma_{1}$	1.136	1.048	1.129	0.006
1011	$\tau, \theta_{\rm A} = \theta_1 = \theta_{2, \rm m} m_{12, \rm \gamma} \gamma_1$	1.148	1.274	0.739	0.021
0023	$\theta_{\rm A} = \theta_1 = \theta_2, \ m_{21}, \ \gamma_1, \ \gamma_2$	1.154	1.311	0.500	0.020

Table 3	Continued
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0230 0321 1000* 1202 0223 1001 0011 0213 1102 1121 1022 1012 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{aligned} \theta_{A} &= \theta_{2},  \theta_{1},  m_{12},  m_{21} \\ \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2},  \gamma_{2} \\ \theta_{A} &= \theta_{2},  \theta_{1},  m_{21},  \gamma_{1},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2},  \gamma_{1} \\ \theta_{A} &= \theta_{1} &= \theta_{2},  m_{12},  \gamma_{1} \\ \theta_{A} &= \theta_{1} &= \theta_{2},  m_{12},  \gamma_{1} \\ \theta_{A} &= \theta_{1},  \theta_{2},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  \theta_{1},  \eta_{2} \\ \eta,  \theta_{A} &= \theta_{A},  \theta_{A} \\ \eta,  \theta_{A} &= \theta_{A} \\ \eta,  \theta_{A} \\ \eta,  \theta_{A} &= \theta_$	1.172 1.173 1.178 1.180 1.181 <b>1.187</b> <b>1.198</b> 1.199 1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280 1.281	1.022 1.106 1.261 1.163 1.173 <b>1.328</b> <b>1.298</b> 1.117 1.217 1.141 <b>1.308</b> <b>1.324</b> 1.159	1.135 1.129 0.971 1.124 1.124 0.752 0.931 1.135 1.129 1.137 1.011 1.129	0.003 0.003 0.015 0.004 0.007 <b>0.021</b> <b>0.022</b> 0.004 0.004 0.004 0.010 <b>0.021</b>
1000* 1202 0223 1001 0011 0213 1102 1121 1022 1012 1332 1322 0212 1312 13	$\begin{aligned} \tau, \theta_{A} &= \theta_{1} = \theta_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, \gamma_{2} \\ \theta_{A} &= \theta_{2}, \theta_{1}, m_{21}, \gamma_{1}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, \gamma_{1} \\ \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{1} \\ \theta_{A} &= \theta_{2}, \theta_{1}, m_{12}, \gamma_{1}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1}, \theta_{2}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1}, \theta_{2}, m_{21}, \gamma_{1} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{1}, \gamma_{2} \end{aligned}$	1.178 1.180 1.181 <b>1.187</b> <b>1.198</b> 1.199 1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280	1.261 1.163 1.173 <b>1.328</b> 1.298 1.117 1.217 1.141 1.308 1.324	0.971 1.124 1.124 <b>0.752</b> <b>0.931</b> 1.135 1.129 1.137 <b>1.011</b>	0.015 0.004 0.007 0.021 0.022 0.004 0.004 0.004
1202 0223 1001 0011 0213 1102 1121 1022 1012 1332 1322 0212 1312 13	$\begin{aligned} \tau, \theta_{A} &= \theta_{1} = \theta_{2}, \gamma_{2} \\ \theta_{A} &= \theta_{2}, \theta_{1}, m_{21}, \gamma_{1}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, \gamma_{1} \\ \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{1} \\ \theta_{A} &= \theta_{2}, \theta_{1}, m_{12}, \gamma_{1}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1}, \theta_{2}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1}, \theta_{2}, m_{21}, \gamma_{1} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A} &= \theta_{1} = \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{2} \\ \eta_{A} &= \theta_{2}, \theta_{1}, m_{12}, \gamma_{2} \\ \tau, \theta_{A}, \theta_{1} &= \theta_{2}, m_{21}, \gamma_{1}, \gamma_{2} \end{aligned}$	1.180 1.181 <b>1.187</b> <b>1.198</b> 1.199 1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280	1.163 1.173 <b>1.328</b> 1.298 1.117 1.217 1.141 <b>1.308</b> <b>1.324</b>	1.124 1.124 <b>0.752</b> <b>0.931</b> 1.135 1.129 1.137 <b>1.011</b>	0.004 0.007 <b>0.021</b> 0.022 0.004 0.004 0.004
0223 1001 0011 0213 1102 1121 1022 1012 1332 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{aligned} \theta_{A} &= \theta_{2},  \theta_{1},  m_{21},  \gamma_{1},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  \gamma_{1} \\ \theta_{A} &= \theta_{1} = \theta_{2},  m_{12},  \gamma_{1} \\ \theta_{A} &= \theta_{2},  \theta_{1},  m_{12},  \gamma_{1},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{1} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{21},  \gamma_{1},  \gamma_{2} \end{aligned}$	1.181 <b>1.187</b> <b>1.198</b> 1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280	1.173 1.328 1.298 1.117 1.217 1.141 1.308 1.324	1.124 <b>0.752</b> <b>0.931</b> 1.135 1.129 1.137 <b>1.011</b>	0.007 0.021 0.022 0.004 0.004 0.004
1001         0011         0213         1102         1121         1022         1012         1332         1322         0212         1312         1323         0123         1003         0313         1433         0312         0211         1320	$\begin{aligned} \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1} = \mathbf{\theta}_{2},  \mathbf{\gamma}_{1} \\ \mathbf{\theta}_{A} &= \mathbf{\theta}_{1} = \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{1} \\ \mathbf{\theta}_{A} &= \mathbf{\theta}_{2},  \mathbf{\theta}_{1},  \mathbf{m}_{12},  \mathbf{\gamma}_{1},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1},  \mathbf{\theta}_{2},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1},  \mathbf{\theta}_{2},  \mathbf{m}_{21},  \mathbf{\gamma}_{1} \\ \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1} = \mathbf{\theta}_{2},  \mathbf{m}_{21},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1} = \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A} &= \mathbf{\theta}_{1} = \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A},  \mathbf{\theta}_{1} &= \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A},  \mathbf{\theta}_{1} &= \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A},  \mathbf{\theta}_{1} &= \mathbf{\theta}_{2},  \mathbf{m}_{21},  \mathbf{\gamma}_{2} \\ \mathbf{\tau},  \mathbf{\theta}_{A},  \mathbf{\theta}_{1} &= \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\gamma}_{1} \\ \mathbf{\tau}_{2},  \mathbf{\tau}_{1},  \mathbf{\theta}_{A},  \mathbf{\theta}_{1} &= \mathbf{\theta}_{2},  \mathbf{m}_{12},  \mathbf{\eta}_{2} \end{aligned}$	<ol> <li>1.187</li> <li>1.198</li> <li>1.205</li> <li>1.211</li> <li>1.214</li> <li>1.270</li> <li>1.271</li> <li>1.280</li> </ol>	1.328 1.298 1.117 1.217 1.141 1.308 1.324	0.752 0.931 1.135 1.129 1.137 1.011	<b>0.021</b> <b>0.022</b> 0.004 0.004 0.010
0011 0213 1102 1121 1022 1012 1332 1322 0212 1312 13	$ \begin{aligned} \theta_{A} &= \theta_{1} = \theta_{2}, \ m_{12}, \ \gamma_{1} \\ \theta_{A} &= \theta_{2}, \ \theta_{1}, \ m_{12}, \ \gamma_{1}, \ \gamma_{2} \\ \tau, \ \theta_{A} &= \theta_{1}, \ \theta_{2}, \ \gamma_{2} \\ \tau, \ \theta_{A} &= \theta_{1}, \ \theta_{2}, \ m_{21}, \ \gamma_{1} \\ \tau, \ \theta_{A} &= \theta_{1} = \theta_{2}, \ m_{21}, \ \gamma_{2} \\ \tau, \ \theta_{A} &= \theta_{1} = \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A} &= \theta_{1} = \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{21}, \ \gamma_{2} \\ \eta_{A} &= \theta_{1}, \ \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{12}, \ \gamma_{2} \\ \tau, \ \theta_{A}, \ \theta_{1} &= \theta_{2}, \ m_{21}, \ \gamma_{1}, \ \gamma_{2} \end{aligned} $	<ol> <li>1.198</li> <li>1.199</li> <li>1.205</li> <li>1.211</li> <li>1.214</li> <li>1.270</li> <li>1.271</li> <li>1.280</li> </ol>	1.298 1.117 1.217 1.141 1.308 1.324	0.931 1.135 1.129 1.137 1.011	<b>0.022</b> 0.004 0.004 0.010
0213 1102 1121 <b>1022</b> <b>1012</b> 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{aligned} \theta_{A} &= \theta_{2},  \theta_{1},  m_{12},  \gamma_{1},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{21},  \gamma_{2} \\ \eta_{A} &= \theta_{2},  \theta_{1},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \end{aligned} $	1.199 1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280	1.117 1.217 1.141 <b>1.308</b> <b>1.324</b>	1.135 1.129 1.137 <b>1.011</b>	0.004 0.004 0.010
1102 1121 <b>1022</b> <b>1012</b> 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$\begin{aligned} \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \eta_{A} &= \theta_{2},  \theta_{1},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  \gamma_{2} \end{aligned}$	1.205 1.211 <b>1.214</b> <b>1.270</b> 1.271 1.280	1.217 1.141 <b>1.308</b> <b>1.324</b>	1.129 1.137 <b>1.011</b>	0.004 0.010
1121 <b>1022</b> <b>1012</b> 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$\begin{aligned} \tau,  \theta_{A} &= \theta_{1},  \theta_{2},  m_{21},  \gamma_{1} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A} &= \theta_{1} = \theta_{2},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{12},  m_{21},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{21},  \gamma_{2} \\ \theta_{A} &= \theta_{2},  \theta_{1},  m_{12},  \gamma_{2} \\ \tau,  \theta_{A},  \theta_{1} &= \theta_{2},  m_{21},  \gamma_{1} \\ \tau_{2} &= \theta_{A},  \theta_{1} &= \theta_{A},  \theta_{A},  \theta_{A} \\ \eta_{A} &= \theta_{A},  \theta_{A},  \theta_{A} &= \theta_{A},  \theta_{A},  \theta_{A} \\ \eta_{A} &= \theta_{A}, $	1.211 1.214 1.270 1.271 1.280	1.141 <b>1.308</b> <b>1.324</b>	1.137 <b>1.011</b>	0.010
<b>1022</b> <b>1012</b> 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$\begin{aligned} \boldsymbol{\tau},  \boldsymbol{\theta}_{A} &= \boldsymbol{\theta}_{1} = \boldsymbol{\theta}_{2},  \mathbf{m}_{21},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A} &= \boldsymbol{\theta}_{1} = \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \mathbf{m}_{21},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{21},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\theta}_{A} &= \boldsymbol{\theta}_{2},  \boldsymbol{\theta}_{1},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \end{aligned}$	<b>1.214</b> <b>1.270</b> 1.271 1.280	1.308 1.324	1.011	
<b>1012</b> 1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$\begin{aligned} \boldsymbol{\tau},  \boldsymbol{\theta}_{A} &= \boldsymbol{\theta}_{1} = \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \mathbf{m}_{21},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{21},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\theta}_{A} &= \boldsymbol{\theta}_{2},  \boldsymbol{\theta}_{1},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \\ \boldsymbol{\tau},  \boldsymbol{\theta}_{A},  \boldsymbol{\theta}_{1} &= \boldsymbol{\theta}_{2},  \mathbf{m}_{12},  \boldsymbol{\gamma}_{2} \end{aligned}$	<b>1.270</b> 1.271 1.280	1.324		0.021
1332 1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{split} \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{12},  m_{21},  \gamma_2 \\ \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{21},  \gamma_2 \\ \theta_A &= \theta_2,  \theta_1,  m_{12},  \gamma_2 \\ \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{12},  \gamma_2 \\ \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{21},  \gamma_1  \gamma_2 \end{split} $	1.271 1.280		1 1 2 9	
1322 0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{split} \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{21},  \gamma_2 \\ \theta_A &= \theta_2,  \theta_1,  m_{12},  \gamma_2 \\ \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{12},  \gamma_2 \\ \tau,  \theta_A,  \theta_1 &= \theta_2,  m_{21},  \gamma_1,  \gamma_2 \end{split} $	1.280	1.159		0.021
0212 1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{split} \theta_A &= \theta_{2,} \; \theta_{1,} \; m_{12,} \; \gamma_2 \\ \tau, \; \theta_{A,} \; \theta_1 &= \theta_{2,} \; m_{12,} \; \gamma_2 \\ \tau, \; \theta_{A,} \; \theta_1 &= \theta_{2,} \; m_{21,} \; \gamma_1, \; \gamma_2 \end{split} $			1.179	0.003
1312 1323 0123 1003 0313 1433 0312 0211 1320	$ \begin{array}{l} \tau, \; \theta_{A_{\text{c}}} \; \theta_{1} \; = \; \theta_{2,} \; m_{12,} \; \gamma_{2} \\ \tau, \; \theta_{A_{\text{c}}} \; \theta_{1} \; = \; \theta_{2,} \; m_{21,} \; \gamma_{1,} \; \gamma_{2} \end{array} $	1 201	1.087	1.233	0.000
1323 0123 1003 0313 1433 0312 0211 1320	$\tau,\;\theta_{A_{\prime}}\;\theta_{1}=\theta_{2_{\prime}}\;m_{21_{\prime}}\;\gamma_{1_{\prime}}\;\gamma_{2}$	1.201	1.181	1.140	0.001
0123 1003 0313 1433 0312 0211 1320		1.286	1.105	1.221	0.001
1003 0313 1433 0312 0211 1320		1.312	1.075	1.239	0.001
0313 1433 0312 0211 1320	$\theta_{\rm A} = \theta_1, \theta_2, m_{21}, \gamma_1, \gamma_2$	1.312	1.189	1.192	0.007
1433 0312 0211 1320	$\tau$ , $\theta_A = \theta_1 = \theta_2$ , $\gamma_1$ , $\gamma_2$	1.321	1.443	1.122	0.007
0312 0211 1320	$\theta_{A_{\prime}} \theta_1 = \theta_{2_{\prime}} m_{12_{\prime}} \gamma_{1_{\prime}} \gamma_2$	1.327	1.207	1.182	0.001
0211 1320	τ, $θ_{A}$ , $θ_{1}$ , $θ_{2}$ , $m_{12}$ , $m_{21}$ , $γ_{1}$ , $γ_{2}$	1.327	0.998	1.269	0.000
0211 1320	$\theta_{A}, \theta_1 = \theta_{2}, m_{12}, \gamma_2$	1.328	1.201	1.209	0.004
1320	$\theta_{\rm A} = \theta_{2}, \theta_{1}, m_{12}, \gamma_{1}$	1.333	1.195	1.256	0.006
	$\tau$ , $\theta_{A_2}$ , $\theta_1 = \theta_2$ , $m_{21}$	1.336	1.235	1.180	0.001
1403	$\tau$ , $\theta_{A}$ , $\theta_{1}$ , $\theta_{2}$ , $\gamma_{1}$ , $\gamma_{2}$	1.350	1.011	1.298	0.000
1330*	$\tau, \theta_{A_{2}}, \theta_{1} = \theta_{2}, m_{12}, m_{21}$	1.351	1.274	1.225	0.006
0323	$\theta_{A_{\ell}} \theta_1 = \theta_{2_{\ell}} m_{21_{\ell}} \gamma_{1_{\ell}} \gamma_2$	1.353	1.170	1.259	0.003
1333	$\tau, \theta_{A}, \theta_{1} = \theta_{2}, m_{12}, m_{21}, \gamma_{1}, \gamma_{2}$	1.357	1.127	1.277	0.003
1103	$\tau, \theta_{A} = \theta_{1}, \theta_{2}, \gamma_{1}, \gamma_{2}$	1.400	1.186	1.408	0.003
1423	$\tau, \theta_{A}, \theta_{1}, \theta_{2}, m_{21}, \gamma_{1}, \gamma_{2}$	1.408	1.502	1.182	0.001
0331	$\theta_{A_{i}} \theta_{1} = \theta_{2_{i}} m_{12_{i}} m_{21_{i}} \gamma_{1}$	1.424	1.314	1.368	0.000
0311	$\theta_{A_{\ell}} \theta_1 = \theta_{2_{\ell}} m_{12_{\ell}} \gamma_1$	1.475	1.353	1.353	0.003
1432	$\tau, \theta_{A}, \theta_{1}, \theta_{2}, m_{12}, m_{21}, \gamma_{2}$	1.500	1.297	1.360	0.000
1402	$\tau, \theta_{A}, \theta_{1}, \theta_{2}, \gamma_{2}$	1.543	1.101	1.545	0.003
0413	$\theta_{A}, \theta_{1}, \theta_{2}, m_{12}, \gamma_{1}, \gamma_{2}$	1.570	1.139	1.545	0.006
0412	$\theta_{A}, \theta_{1}, \theta_{2}, m_{12}, \gamma_{2}$	1.575	1.172	1.516	0.001
0322	$\theta_{A}, \theta_{1} = \theta_{2}, m_{21}, \gamma_{2}$	1.591	1.493	1.481	0.001
1303	$\tau$ , $\theta_{A_1}$ , $\theta_1 = \theta_2$ , $\gamma_1$ , $\gamma_2$	1.591	1.303	1.610	0.003
1301	$\tau, \theta_{A}, \theta_{1} = \theta_{2}, \gamma_{1}$	1.621	1.428	1.554	0.001
1300*	$\tau, \theta_{A}, \theta_1 = \theta_2$	1.630	1.342	1.562	0.004
1313	$\tau, \theta_{A_{1}} \theta_{1} = \theta_{2}, m_{12}, \gamma_{1}, \gamma_{2}$	1.676	3.419	1.164	0.007
0423	$\theta_{A}, \theta_{1}, \theta_{2}, m_{21}, \gamma_{1}, \gamma_{2}$	1.710	1.358	1.593	0.000
0430	$\theta_{A}, \theta_{1}, \theta_{2}, m_{21}, m_{1}, m_{21}$	1.715	1.294	1.620	0.000
0113	$\theta_{A}, \theta_{1} = \theta_{2}, m_{12}, \gamma_{1}, \gamma_{2}$	1.715	5.727	1.068	0.004
0411	$\theta_{A}, \theta_{1}, \theta_{2}, m_{12}, \gamma_{1}$	1.717	1.259	1.665	0.003
0422	$\theta_{A}, \theta_{1}, \theta_{2}, m_{21}, \gamma_{2}$	1.759	1.417	1.614	0.000
1401	$\tau_{A}, \theta_{1}, \theta_{2}, \eta_{2}$ $\tau_{L}, \theta_{A}, \theta_{1}, \theta_{2}, \gamma_{1}$	1.781	1.835	1.505	0.001
0433	$\theta_{A}, \theta_{1}, \theta_{2}, m_{12}, m_{21}, \gamma_{1}, \gamma_{2}$	1.843	1.773	1.597	0.000
0021	$\theta_{\rm A} = \theta_1 = \theta_2, m_{21}, \gamma_1$	1.867	4.813	0.673	0.000
0221	$\theta_{\rm A} = \theta_1 - \theta_2, \ m_{21}, \ \gamma_1$ $\theta_{\rm A} = \theta_2, \ \theta_1, \ m_{21}, \ \gamma_1$	1.934	6.915	0.937	0.006
1400	$\sigma_{A} = \sigma_{2}, \sigma_{1}, m_{21}, \gamma_{1}$ $\tau, \sigma_{A}, \sigma_{1}, \sigma_{2}$	2.098	1.697	1.899	0.000
0232	$\theta_{A} = \theta_{2}, \theta_{1}, m_{12}, m_{21}, \gamma_{2}$	2.186	7.859	1.121	0.007
0232	$\theta_{\rm A} = \theta_{2},  \theta_{1},  m_{12},  m_{21},  \gamma_{2}$ $\theta_{\rm A} = \theta_{1},  \theta_{2},  m_{21},  \gamma_{2}$	2.356	7.532	1.121	0.007
1122	$v_A = v_1 v_2 m_{21} v_2$	2.556	8.798	1.234	
		2.001	0./90		111112
1133 1410	$\begin{aligned} & \tau_{A} = \theta_{1}, \ \theta_{2}, \ m_{21}, \ \gamma_{2} \\ & \tau_{7}, \ \theta_{A} = \theta_{1}, \ \theta_{2}, \ m_{21}, \ \gamma_{2} \\ & \tau_{7}, \ \theta_{A} = \theta_{1}, \ \theta_{2}, \ m_{12}, \ m_{21}, \ \gamma_{1}, \ \gamma_{2} \end{aligned}$	2.748	12.927	0.814	0.003 0.008

Table 3 (	Continued
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Model	Parameters	Mean	SD	Median	Posterior probability
1420	$\tau$ , $\theta_{A}$ , $\theta_{1}$ , $\theta_{2}$ , $m_{21}$	2.819	9.142	1.557	0.001
0330	$\theta_{A_\ell}$ $\theta_1 = \theta_{2_\ell}$ $m_{12_\ell}$ $m_{21}$	3.156	11.980	1.608	0.000
0431	$\theta_{A_{2}}$ , $\theta_{1_{2}}$ , $\theta_{2_{2}}$ , $m_{12_{2}}$ , $m_{21_{2}}$ , $\gamma_{1}$	3.388	12.338	1.687	0.001
0432	$\theta_{A}, \theta_{1}, \theta_{2}, m_{12}, m_{21}, \gamma_{2}$	3.769	15.818	1.606	0.003
1210	$\tau$ , $\theta_A = \theta_2$ , $\theta_1$ , $m_{12}$	4.007	21.699	0.880	0.010
0310	$\theta_{A_{\ell}}$ $\theta_1 = \theta_{2_{\ell}}$ $m_{12}$	4.405	20.648	1.670	0.001
0421	$\theta_{A_{\ell}}$ $\theta_{1_{\ell}}$ $\theta_{2_{\ell}}$ $m_{21_{\ell}}$ $\gamma_{1}$	4.761	18.586	1.563	0.000
1223	$\tau, \theta_{\rm A} = \theta_2, \theta_1, m_{21}, \gamma_1, \gamma_2$	4.813	27.942	0.880	0.007
0410	$\theta_{A_{\ell}}$ $\theta_{1_{\ell}}$ $\theta_{2_{\ell}}$ $m_{12}$	4.840	19.483	1.684	0.000
0333	$\theta_{A_{1}}$ , $\theta_{1} = \theta_{2_{1}}$ , $m_{12_{1}}$ , $m_{21_{1}}$ , $\gamma_{1_{1}}$ , $\gamma_{2}$	4.841	24.764	1.304	0.004
1411	$\tau, \theta_{A}, \theta_{1}, \theta_{2}, m_{12}, \gamma_{1}$	4.949	22.725	1.182	0.000
0320	$\theta_{A}, \theta_1 = \theta_2, m_{21}$	5.184	25.275	1.771	0.000
1431	$\tau, \theta_{A_{\ell}}, \theta_{1_{\ell}}, \theta_{2_{\ell}}, m_{12_{\ell}}, m_{21_{\ell}}, \gamma_{1}$	5.539	28.987	1.440	0.000
1421	$\tau, \theta_{A}, \theta_{1}, \theta_{2}, m_{21}, \gamma_{1}$	5.618	22.805	1.418	0.001
1311	$\tau$ , $\theta_{A_{\ell}}$ , $\theta_1 = \theta_{2_{\ell}}$ , $m_{12_{\ell}}$ , $\gamma_1$	5.721	32.177	1.137	0.001
0111	$\theta_{\rm A} = \theta_{1},  \theta_{2},  m_{12},  \gamma_1$	5.804	32.950	1.143	0.008
0420	$\theta_{A}, \theta_{1}, \theta_{2}, m_{21}$	6.037	28.946	1.629	0.001
1412	$\tau$ , $\theta_{A}$ , $\theta_{1}$ , $\theta_{2}$ , $m_{12}$ , $\gamma_{2}$	6.186	23.177	1.611	0.003
0010	$\theta_{\rm A} = \theta_1 = \theta_2, \ m_{12}$	6.223	36.293	0.000	0.017
1413	$\tau$ , $\theta_{A}$ , $\theta_{1}$ , $\theta_{2}$ , $m_{12}$ , $\gamma_{1}$ , $\gamma_{2}$	8.209	48.083	1.344	0.000
1430	$\tau, \theta_{A_{\ell}}, \theta_{1_{\ell}}, \theta_{2_{\ell}}, m_{12_{\ell}}, m_{21}$	8.661	50.499	1.516	0.001
1422	$\tau$ , $\theta_{A}$ , $\theta_{1}$ , $\theta_{2}$ , $m_{21}$ , $\gamma_{2}$	9.269	45.089	1.344	0.006
0121	$\theta_{\rm A} = \theta_{1}, \theta_{2}, m_{21}, \gamma_{1}$	9.369	56.607	1.327	0.004
1302	$\tau$ , $\theta_{A}$ , $\theta_{1} = \theta_{2}$ , $\gamma_{2}$	9.386	44.243	1.233	0.004
0120	$\theta_{\rm A} = \theta_1,  \theta_2,  \mathbf{m}_{21}$	9.466	57.924	1.189	0.004
1310	$\tau$ , $\theta_{A}$ , $\theta_{1} = \theta_{2}$ , $m_{12}$	9.812	60.333	1.206	0.000
1100	$\tau,  \theta_{\rm A} = \theta_1,  \theta_2$	10.795	68.438	1.121	0.007
0332	$\theta_{A_{\prime}}$ $\theta_{1} = \theta_{2_{\prime}}$ $m_{12_{\prime}}$ $m_{21_{\prime}}$ $\gamma_{2}$	13.053	82.999	1.415	0.004
1120	$\tau, \theta_{\rm A} = \theta_1, \theta_2, m_{21}$	14.667	54.818	1.365	0.007
X0X1*	$\theta_{A}, \gamma_{1}$	16.013	5.576	15.576	0.000
X0X0*	$\theta_{A}$	17.048	7.013	16.115	0.000
0000	$\theta_A = \theta_1 = \theta_2$	116.825	42.505	116.338	0.000

Model numbering scheme is followed by the parameters included in that model (see Fig. 2). They are ranked from lowest to highest MED including the SD and median. The initial five-model ABC test models are indicated by an asterisk '\*'. The 22 models with PP above the random expectation from the 143-model test are bolded.

(albeit similar) isolation models that include some type of migration and population expansion using ABC. This result may be explained by parameter estimates made under similar models. Parameter estimates of  $\tau$  under the optimal model (Table 2) place divergence between the northern and southern populations at approximately 300 000 generations before present, with gene flow from the southern to the northern populations ( $m_{21} = 2.858$ ). Each population experiences expansion, but the rate is greater in the north (3.776) than in the south (1.678). Similar parameter values are estimated using models such as 1032 and 1021.

#### Posterior predictive simulation

Posterior predictive simulation was conducted to assess the nonrelative fit between the various models and the

empirical data. MEDs of all models are shown in Table 3 (see also Fig. S1, Supporting Information). Two results are notable: the models with the highest PP generally have low MED scores, thus indicating that the data generated from the posterior distribution of these models are a close match to the empirical data. However, there is no significant correlation ( $R^2 = 0.02$ ; P = 0.07; Appendix S2, Supporting Information) between PP and MED. Furthermore, some models (i.e. model 0010) that have comparatively poor MED scores nevertheless exhibit PP that exceeds the prior expectation (0.007). This highlights the stochasticity inherent to ABC; with a large number of models and a prior distribution of finite size, some parameter draws from some models will occasionally generate data that are similar to the target, even if the model is not a close match to the true model. While this assessment is one justification for the posterior predictive simulations, another is

**Table 4** Results of the nested model comparison are shown, as well as the final comparison. Models with the highest PP are highlighted in bold

Model	PP	BF
Divergence models		
1001	0.07	2.6
1011	0.04	4.5
1012	0.05	3.6
1013	0.05	3.6
1021	0.11	1.6
1022	0.11	1.6
1023	0.18	1
1030	0.07	2.6
1031	0.07	2.6
1032	0.09	2.0
1033	0.14	1.3
Island models		
0010	0.08	2.3
0011	0.08	2.3
0012	0.12	1.5
0013	0.04	4.5
0020	0.10	1.8
0022	0.12	1.5
0023	0.06	3.0
0030	0.08	2.3
0031	0.08	2.3
0032	0.04	4.5
0033	0.18	1
Final comparison		
1023	>0.9	
0033	<0.1	

that the MED values allow additional evaluation of the importance of different classes of parameters.

Models were partitioned into parameter classes before comparing the mean of the MEDs per parameter class (Fig. S1, Supporting Information). For example, when models are grouped into island, panmictic and isolation models, the isolation model has a mean MED (2.39) far lower than that of the panmictic (16.53) and island (4.30) models. Similarly, models with no change in  $\theta$  have lower MED (1.28) than those that include changes in this parameter (1.43 and 2.86 for 1 change in  $\theta$ ; 2.97 and 3.56 for changes in both). The relationship between gene flow and population expansion appears to be more complex. Models with migration parameterized either in one or in both directions have MEDs (4.47, 2.79 and 2.046) that are all much better than models without migration (7.67). When models were grouped by the population expansion parameters, the average MEDs were similar so long as migration was also included in the model (1.72 for expansion in each population, compared to 6.49 for expansion without migration and 2.24 or 2.32 for expansion in only one population). This supports the idea that several models, similar in their parameterization, are reasonable for the empirical system.

Because it is also reasonable to speculate that the PP of the true model would be positively correlated with the level of differentiation among models in the comparison set, we conducted an analysis where the difference among models was enumerated (i.e. model 0000 and model 1000 were more similar than model 0000 and model 1234 due to similarity of 3/4 parameter classes) and compared to the PP of the generating model. Results do not indicate that such a correlation exists ( $R^2 < 0.01$ , P = 0.88), suggesting that a more subtle interaction among the model types and assumed parameter values contributes towards the accuracy in identifying the true models in the simulation testing.

Density plots for each summary statistic from the ABC vector were plotted individually for three models (0033, 1000 and 1023) using R 2.15.1 to assess model adequacy and any bias in the summary statistics used for the ABC analysis (Fig. 6). Each summary statistic for the three models shares a similar distribution and the empirical estimates are well within this distribution. If the summary statistics were biased, we would expect the distribution for a given summary statistic to be different under a different model. If the models could not somehow represent the data, the empirical summary statistic would fall outside the PPS distribution. Furthermore, the correlation between the number of parameters and the PP ( $R^2 = 0.06$ , P = 0.002) and between the number of parameters and the MED ( $R^2 = 0.04$ , P = 0.01) is extremely weak; this bolsters the argument of Beaumont et al. (2010) that the dimensionality of models does not influence the calculation of the PP in phylogeographic model comparison.

#### Conclusions

Our ABC model selection procedure enabled us to identify a demographic model that is both a good fit to the data (MED<sub>1023</sub> = 0.956; final  $PP_{1023} > 0.9$ ) and consistent with known facts regarding the geologic history of the inland temperate rain forest. Plethodon idahoensis has occupied this region for millions of years (Carstens et al. 2005), but recurrent glaciation during the Pleistocene forced the species into two refugia located within the clearwater river drainage. Like many other temperate species (Hewitt 2004), available data indicate that the Pleistocene climatic fluctuations had a substantial impact on the population genetic structure of P. idahoensis. Our results imply strongly that these refugia were separated latitudinally (i.e. into northern and southern refugia) and that this separation is responsible for the northern-southern population genetic structure observed here. We also



**Fig. 6** Density plots for each summary statistic in the vector used for all analyses for the following models. Model 0033: island with no change in  $\theta$ , migration in both directions and expansion in both populations. Model 1000: isolation model with no change in  $\theta$ , no migration and no expansion. Model 1023: isolation model with no change in  $\theta$ , migration from population 1 to 2 and expansion in both populations. Black arrows indicate where the empirical estimate falls.

find support for population expansion, particularly in the northern populations, and the results suggest that gene flow likely occurs over the ridge separating the Lochsa and Selway rivers in central Idaho.

Researchers have been reluctant to adopt demographic model choice with ABC as a tool for phylogeographic inference, and there are few empirical examples of investigations that rely on this approach. It is unlikely that this reluctance is due to a mistrust of ABC methods in general, or to latent concerns about the applicability of the methods, as many researchers have utilized msBayes, arguably a more complex and (due to the requirement of comparative data) less applicable method developed by Hickerson et al. (2007). Rather, researchers in nonmodel systems may find it difficult to parameterize models due to a lack of prior information, and thus are hesitant to rely only on their intuition to develop a prior set of models to analyse. Model choice with ABC offers a great deal of promise to phylogeographic investigations in nonmodel systems, but only if the models in the comparison set can be identified in a systematic (and nonbiased) manner. We illustrate here that the posterior probabilities in these comparisons are dependent on the composition of the model set, and develop an approach for identifying models for inclusion in a model set that allows a wide range of models to be considered. We first parameterized a large set of possible models, then conducted a preliminary comparison of all models, before selecting only those models with a greater PP than expected by chance for inclusion in the final comparison. PPS was used to check for model adequacy and bias in summary statistics. While this approach may be criticized as being ad hoc, it is decidedly less so than one that only considers models proposed by previous work or chooses them based only on the intuition of researchers; all models in such sets could be poor (as in our empirical example) but one could nevertheless receive a high relative posterior probability. Our work demonstrates that careful consideration of the composition of the model set is vital to ABC model choice experiments and that more attention should be devoted to this issue.

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#### Data accessibility

Sequences for all genes have been deposited in GenBank under the following Accession nos. JX978543-JX978577 and in DRYAD doi:10.5061/dryad.8kq65. Model prior and scripts can also be found at doi:10.5061/dryad.8kq65.

# Supporting information

Additional supporting information may be found in the online version of this article.

**Appendix S1** Replicate 5-model ABC results; R package ABC cross-validation with subset of models.

Appendix S2 Power analysis and regression information.

Fig S1. PP and MED on partitioned groups of models.

 Table S1 Primer and PCR conditions.

Table S2 Summary statistic testing.

Table S3 ABC regression and threshold testing.